Chapter 13
Filtering and Resolution (part 1)

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- **Previous classes:**
  - Spatial encoding (Chaps. 9 & 10)
  - DFT (Chap. 12)
- **Today's content**
  - Review FT reconstruction
  - Filters and Point Spread Functions
  - Gibbs Ringing
  - Q&A

FT image reconstruction

- Spatial encoded signal and spin density

Continuous:

$$s(k_x, ky, kz) = \iiint dxdydz\, \rho(x, y, z) e^{-i2\pi(k_x x + k_y y + k_z z)}$$

$$\rho(x, y, z) = \iiint dk_x dk_y dk_z\, s(k_x, k_y, k_z) e^{i2\pi(k_x x + k_y y + k_z z)}$$

Infinite discrete sampling:

$$\hat{\rho}_\infty(x) = \Delta k \sum_{p=-\infty}^{\infty} s(p\Delta k) e^{i2\pi p\Delta k x}$$

Finite discrete sampling:

$$\hat{\rho}_w(x) = \Delta k \sum_{p=-n}^{n-1} s(p\Delta k) e^{i2\pi p\Delta k x}$$

$$= \hat{\rho}_\infty(x) * W sinc(\pi W x) e^{-i\pi x \Delta k} \quad (W = N\Delta k)$$

Ideal voxel size:

$$\Delta x = \frac{L}{N} = \frac{1}{N\Delta k} = \frac{1}{W}$$

Filters and Point Spread Function (PSF)

- Filter: H(k) multiplies with k-space
- PSF: iFT of H(k), i.e. h(r), convolve with the image

- According to Convolution Theorem:

$s_H(k) = s(k) \cdot H_1(k) \cdot H_2(k)$ ..... --> $\rho_H(x) = \rho(x) * h_1(x) * h_2(x)$...

- Examples of filter:
  - K-space truncation (HP filter)
  - Discrete sampling (mild LP filter)
  - T2/T2' relaxation (LP filter)

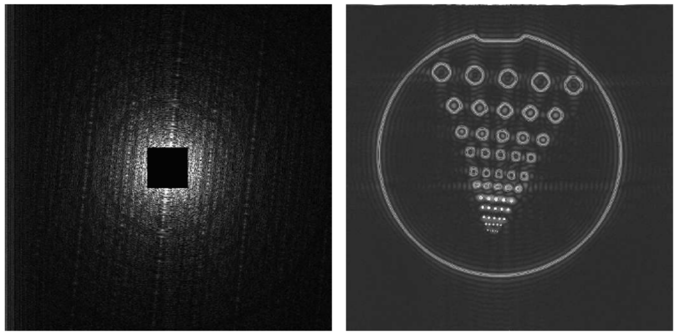

(a) (b)

Fig.13.4

Gibbs Ringing

- Takes place at step discontinuities
- Oscillating over- and under-shoots, with constant amplitude limit

$$\lim_{N\to\infty}\left|\hat{\rho}\left(x_0^{\pm}\right) - \rho\ (x_0^{\pm})\right| \approx 0.09|\rho\ (x_0^{+}) - \rho\ (x_0^{-})|$$

- Oscillating frequency (spatial) depends only on pixel number away from the discontinuity

Gibbs Ringing

- Discontinuity in the object

$$f(x) = fc\,(x) + (f(0^+) - f(0^-))\Theta(x)$$

- Symmetric sampling window on k-space

$$H_w^{sym}(k) = rect(k/W) => h_w^{sym}(x) = Wsinc(\pi W x)$$

- Reconstructed image

$$\hat{f}(x) = f(x) * h_w^{sym}(x)$$

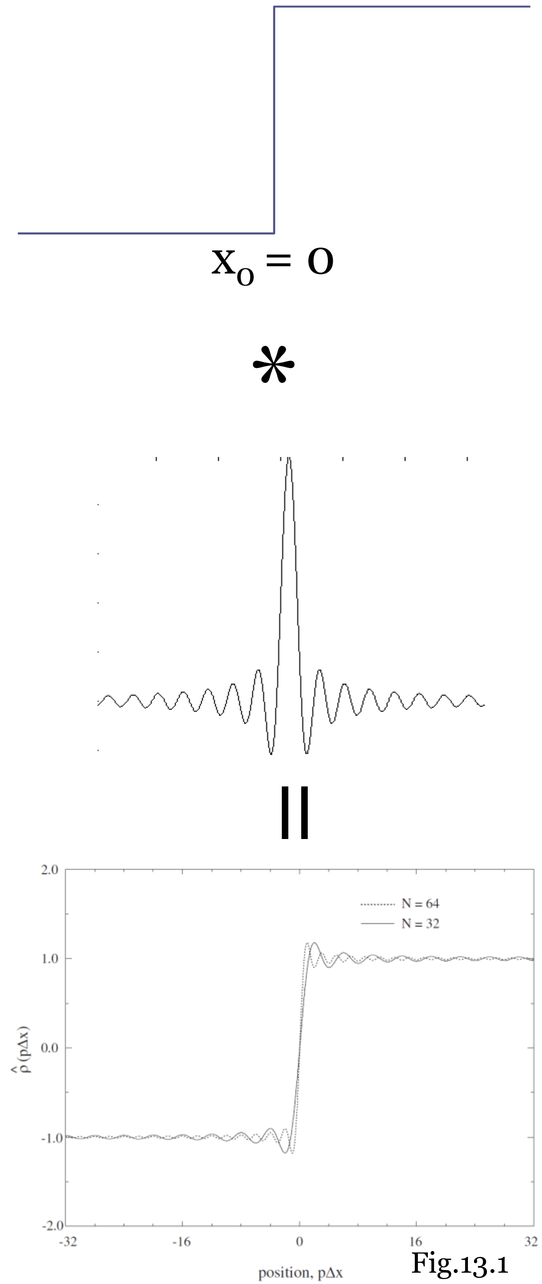


Fig.13.1

Gibbs Ringing: oscillation amplitude limit

$$\hat{f}(x) = f(x) * h_w^{sym}(x)$$

- For fixed FOV and N → ∞, one has:

$$W = \frac{1}{\Delta x} \to \infty$$

- The limit of $\hat{f}(x)$ is:

$$\lim_{W\to\infty} \hat{f}(x) = f_c(x) \lim_{W\to\infty} \int_{-\infty}^{\infty} h_w^{sym}(x - x')dx' + \left(f(0^+) - f(0^-)\right) \lim_{W\to\infty} \int_{x'=0}^{\infty} h_w^{sym}(x - x')dx'$$

$$= f_c(x) + \left(f(0^+) - f(0^-)\right) \lim_{W\to\infty} \int_{x_0=0}^{\infty} h_w^{sym}(x - x')dx'$$

$$= fc(x) + \left(f(0^+) - f(0^-)\right) \lim_{W\to\infty} \left(\frac{1}{2} + \frac{1}{\pi} Si(\pi W x)\right)$$

- The largest oscillation takes place at Δx, so that

$$Si(\pi W \Delta x) = Si(\pi) \cong 1.8519$$

thus

$$\lim_{W\to\infty} \hat{f}(\Delta x) \cong f(\Delta x) + 0.09(f(0^+) - f(0^-))$$

Gibbs Ringing: oscillation frequency

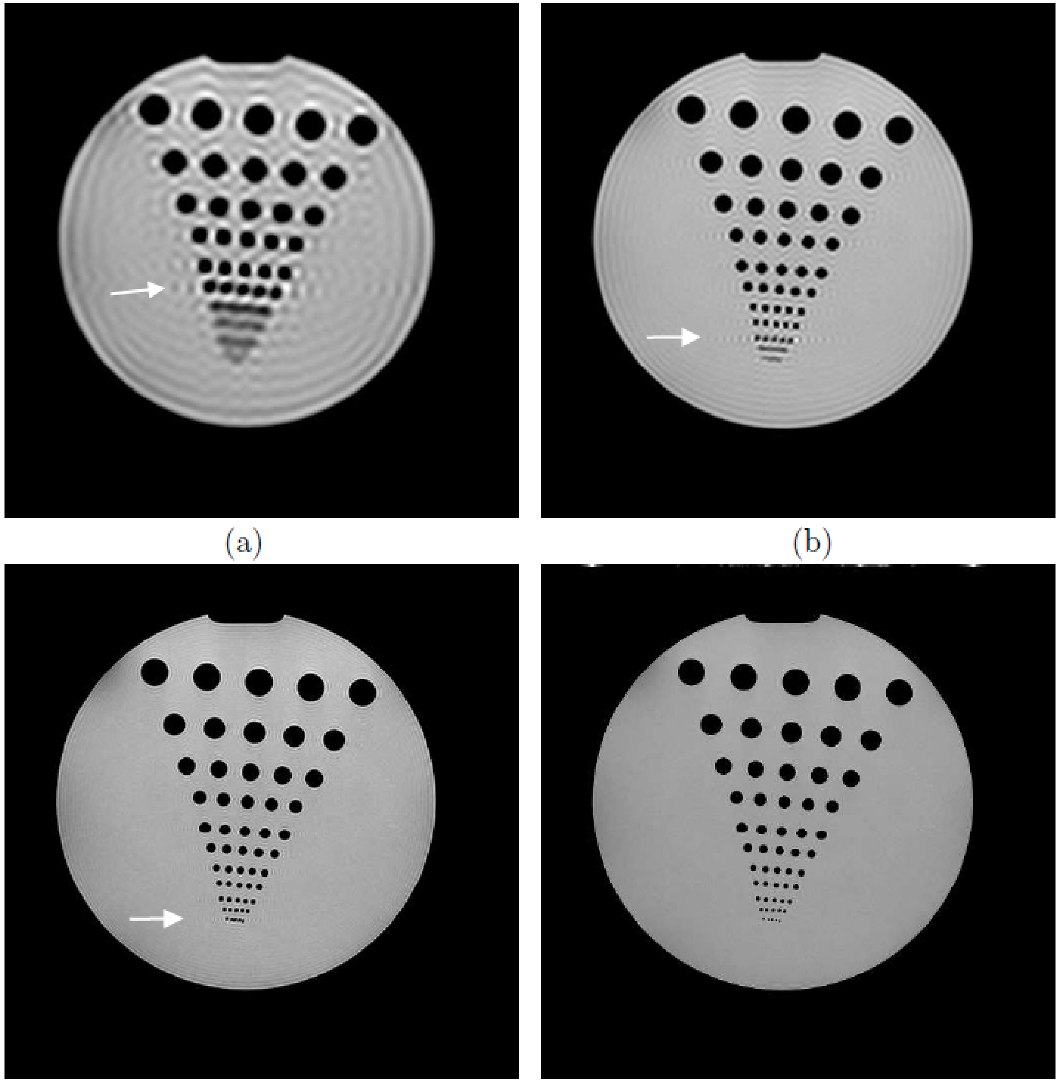
- $\hat{f}(x) = f(x) * h_w^{sym}(x)$
  $= f(x) * W sinc(\pi W x)$
  $= f(x) * W sinc\left(\frac{\pi x}{\Delta x}\right)$
  $= f(x) * W sinc(q\pi)$

- Spatial frequency of the Gibbs Ringing depends only on pixel size Δx

- Higher resolution -> smaller pixels -> denser Gibbs Ringing but slightly greater signal variation

Gibbs Ringing: examples

- Note the coherent addition of the ringing



(a) (b) (c) (d)

Fig.13.3

Gibbs Ringing: reduction

- Properties of Gibbs Ringing
  - Proportional to the signal difference at the discontinuity
  - Ringing variation in amplitude and frequency is a function of pixel number
  - Over- and under-shoot alternates by every other pixel
  - The less sharper the discontinuity, the less obvious the Gibbs Ringing
- Gibbs Ringing reduction
  - LP filter (e.g. Hanning, Hamming, Gaussian) to smooth out the image

Homework

- Prob 13.1

Next Class

Chapter 13.4 – 13.5

Chapter 13
Filtering and Resolution (part 2)

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- **Previous classes:**
  - Spatial encoding (Chaps. 9 & 10)
  - DFT (Chap. 12)
  - Filters and Point Spread Functions (Chap.13)
  - T2 and T2* decay (chap.8)
- **Today's content**
  - Spatial resolution in MR
  - Filtering due to T2/T2* decay
  - Intro for partial Fourier imaging
  - Review previous sessions

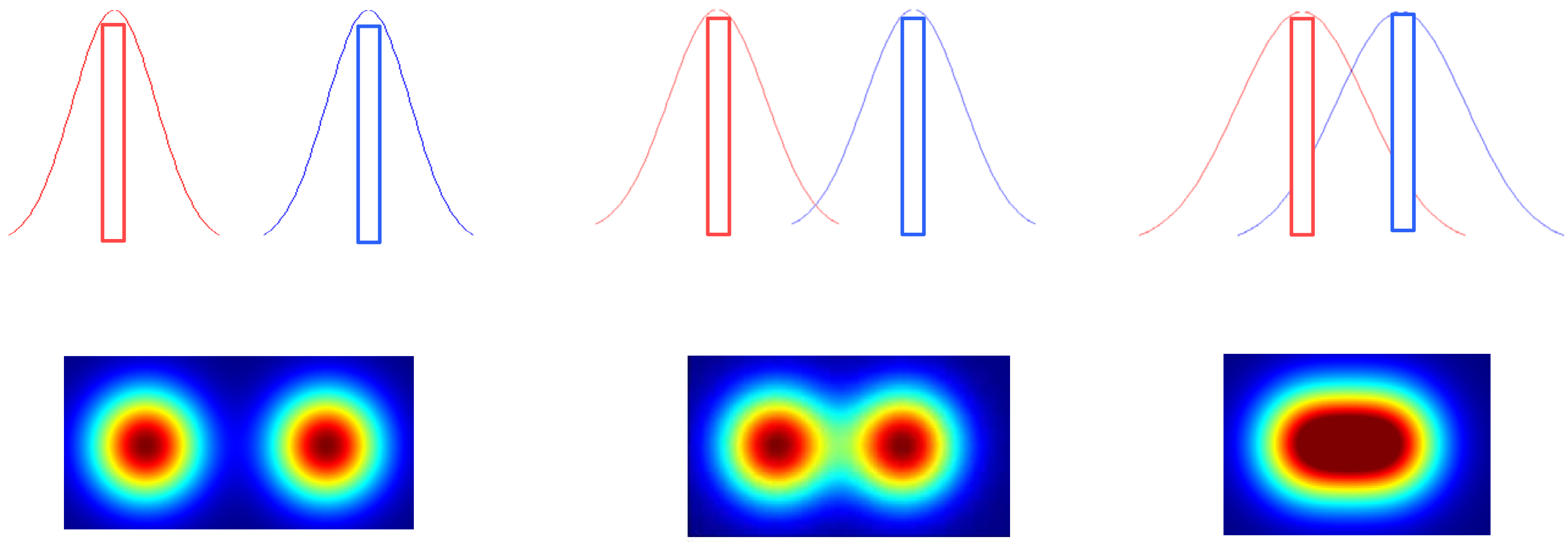
Spatial Resolution in MRI

- Definition

The smallest resolvable distance between two different objects/features

- PSF (if known) can be used to quantify resolution limit

Ideal case: PSF = delta function



PSF and spatial resolution

- Spatial resolution of PSF (continuous filter)

$$\Delta x_{filter} \equiv \frac{1}{h_{filter}(0)} \int_{-\infty}^{\infty} dx h_{filter}(x) = \frac{H_{filter}(0)}{h_{filter}(0)}$$

- Spatial resolution of discrete, windowed, sampled MR signal

$$\Delta x_{MRI,filter} \equiv \frac{1}{h_{ws,filter}(0)} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx\ h_{ws,filter}(x)$$

$$\Downarrow \text{ w/o additional filter}$$

$$\Delta x_{MRI,ws} = \frac{1}{h_{ws}(0)} \int_{-\frac{L}{2}}^{\frac{L}{2}} dx\ h_{ws}(x) = \frac{1}{2n\Delta k} = \Delta x_{FT}$$

- Conditions:
  - H_{filter} is symmetric
  - $H_{filter}(0)$ $H_{filter}(k$ $0)$

K-space coverage and spatial resolution

$$\frac{1}{2n\Delta k} = \frac{1}{W} = \Delta x = \frac{L}{2n}$$

- (fixed L)
  2n
  -> Δx
  -> Gibbs Ringing
  -> SNR

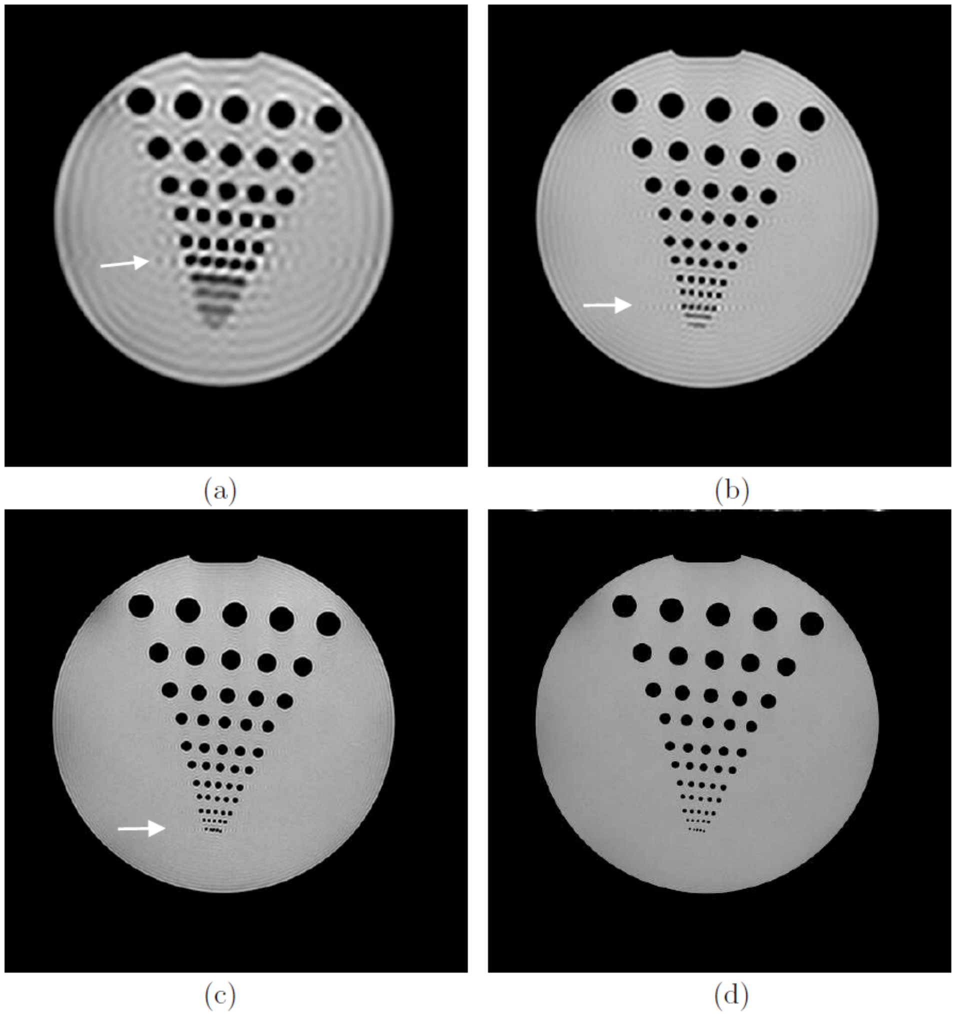
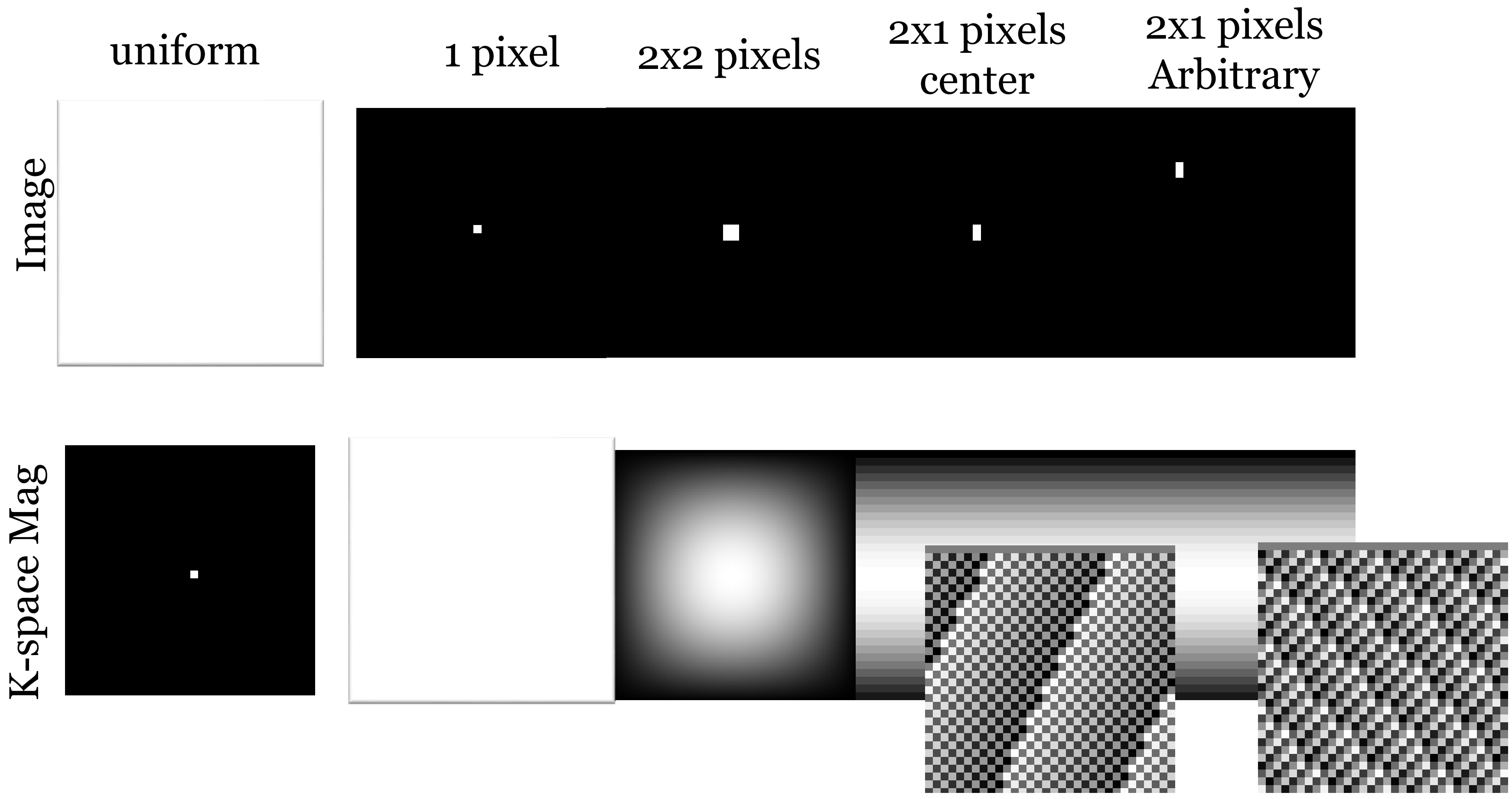


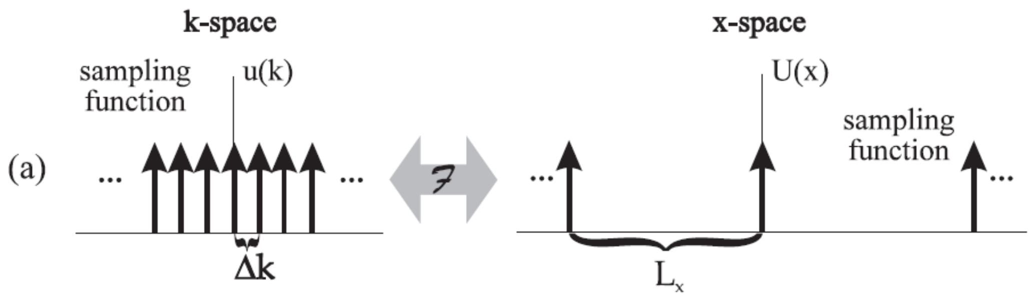
Fig.13.3

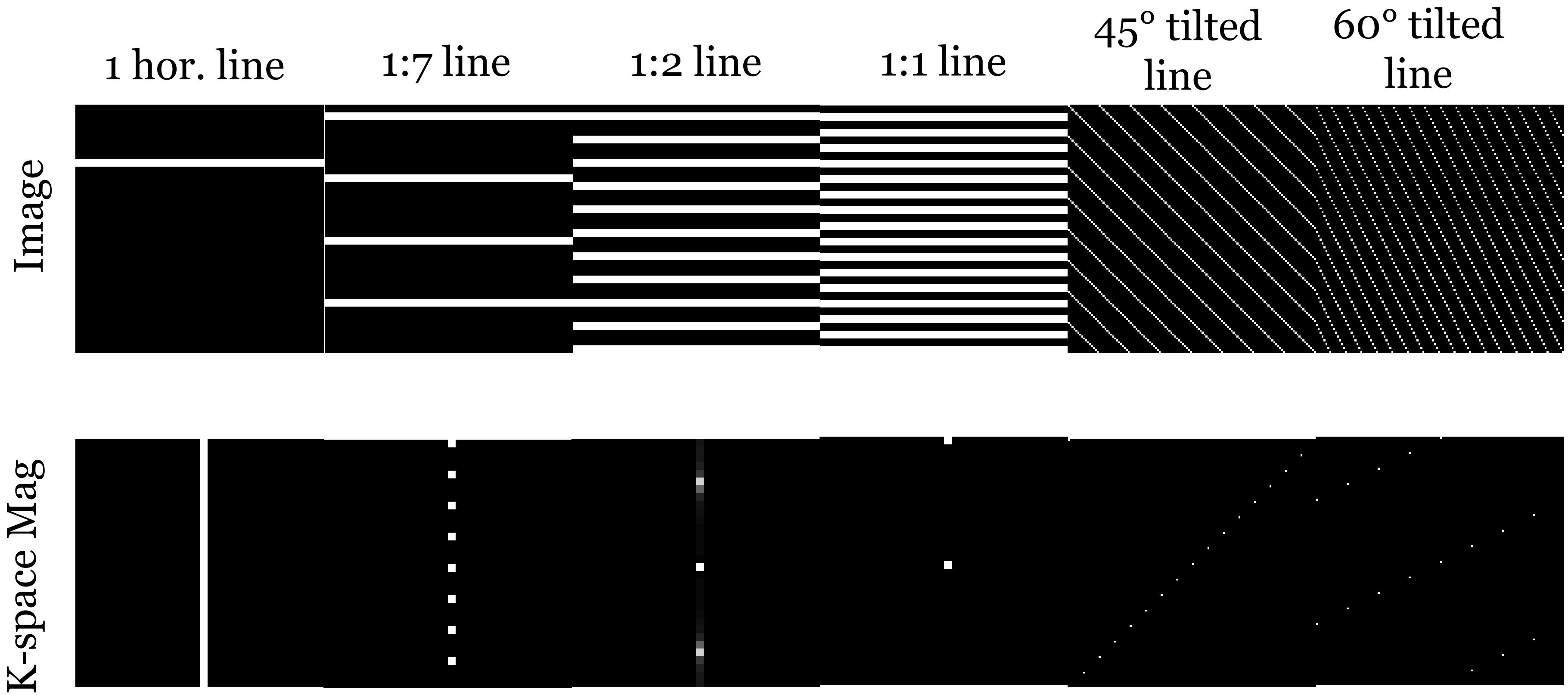
Information content in k-space

- K-space center: low spatial frequency components
- K-space edge : high spatial frequency components



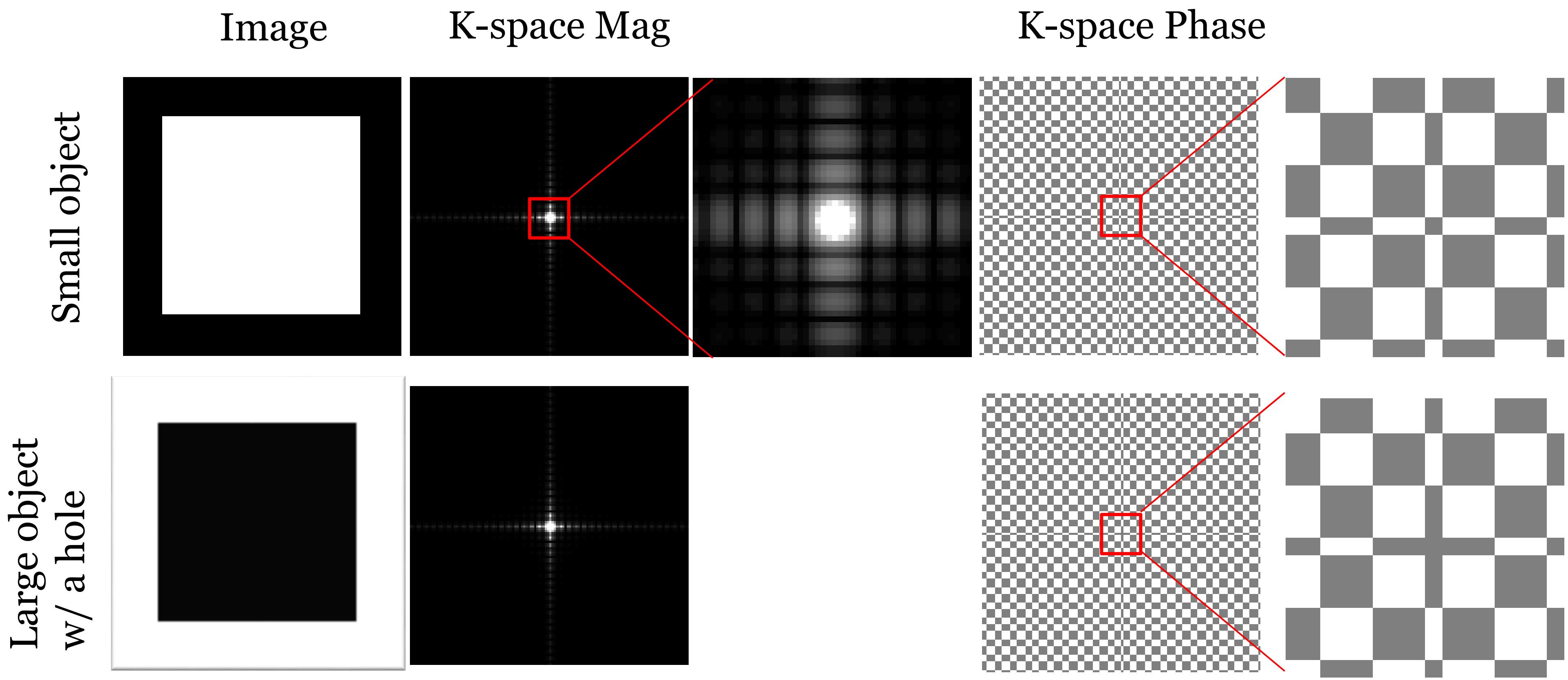
Information content in k-space





Information content in k-space

- K-space phase



Other measures of resolution

- First zero crossing of the filter h(x), e.g. Sinc

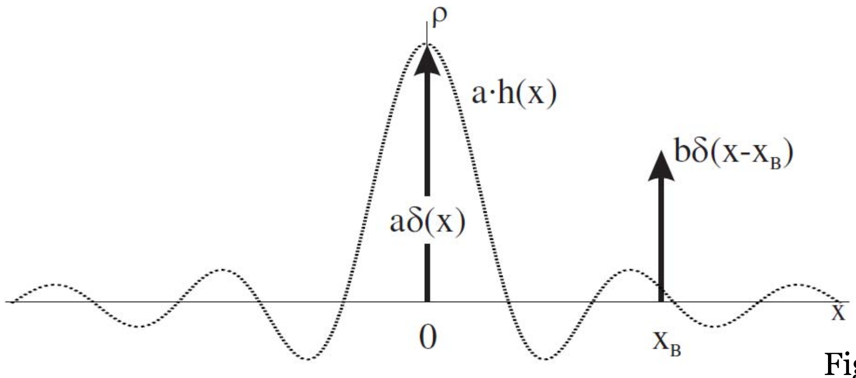


Fig.13.5

- Full Width Half/Tenth Maximum approximation

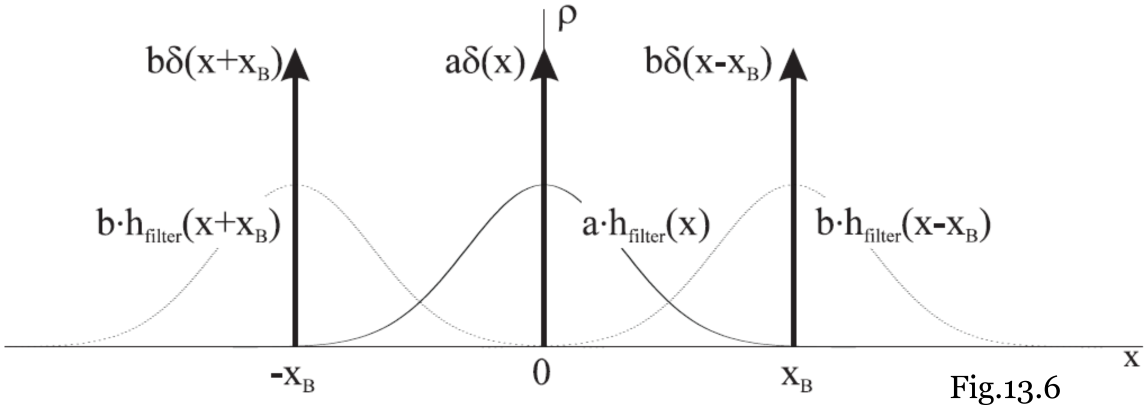
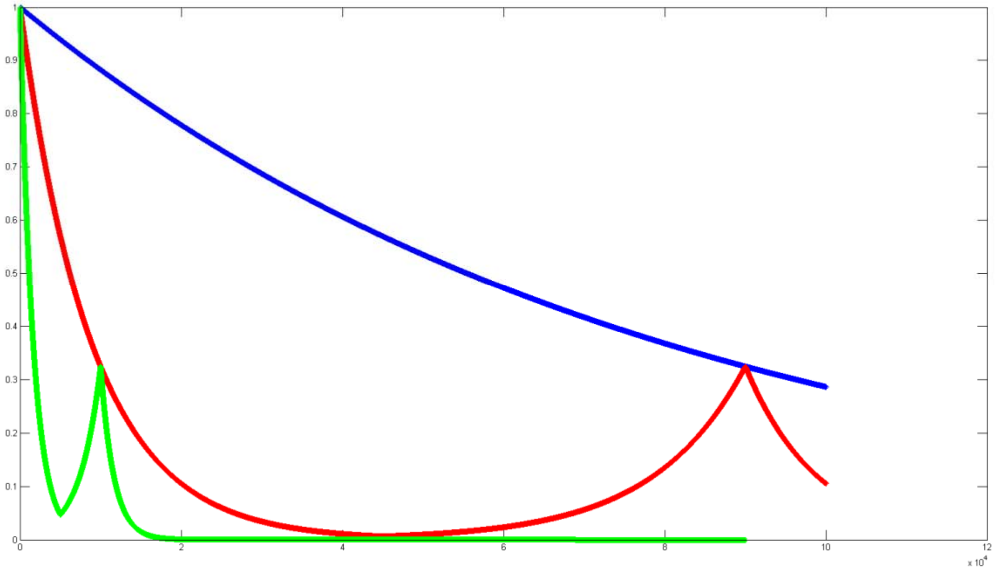


Fig.13.6

Filtering due to T2 and T2* decay



- Posing an intrinsic resolution limit for MR images
- Depend on sequence and scanning parameters in a complicated way (Chap. 15)

Filtering due to T2 and T2* decay

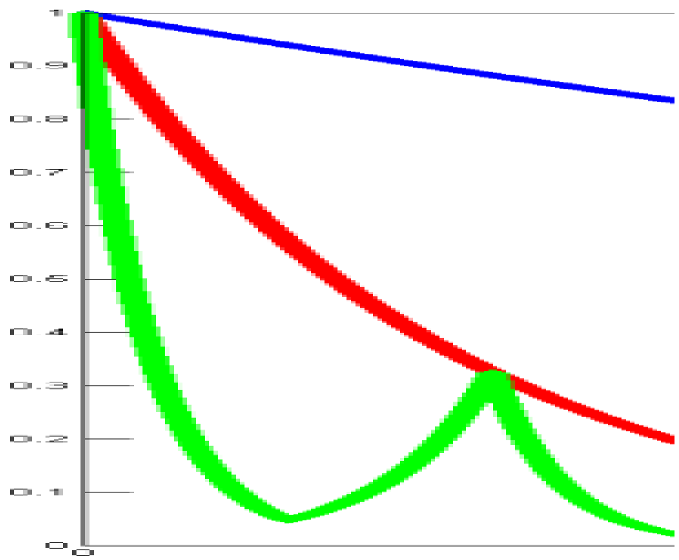
- Along Read direction

$$H_{w(sym),T_2^*}(k) = e^{-\frac{\frac{k}{\gamma G}+TE}{T_2^*}} Rect(\frac{k}{W})$$

iFT is Lorentzian function

iFT is Sinc function

$$h_{w(sym),T_2^*}(x) = Eq\ (13.54)$$



- When in effect?
  - Long RO relative to T2 or T2*

- Pros and Cons
  - Limit the effective resolution, even blur the image
  - Reduce Gibbs ringing

Zero filled interpolation

- Reduces image pixel size

$$\Delta x_{pixel} = 1/N_{img}\Delta k$$

- Does not change the Fourier transform pixel size (i.e. the spatial resolution with limit from sampling)

$$\Delta x_{FT} = 1/N\Delta k$$

- Can recover signal loss for sub-voxel sized objects

- Usually zero pad k-space to a size of power of 2

Homeworks

- Prob *13.5*, *13.7*, *13.8*

Next Class

- Midterm (better bring a pencil and eraser)